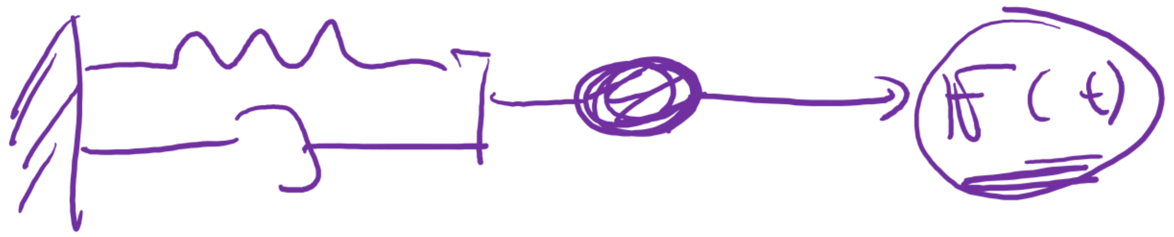
Semaine 4
Doutes

ME-332 – Mécanique Vibratoire

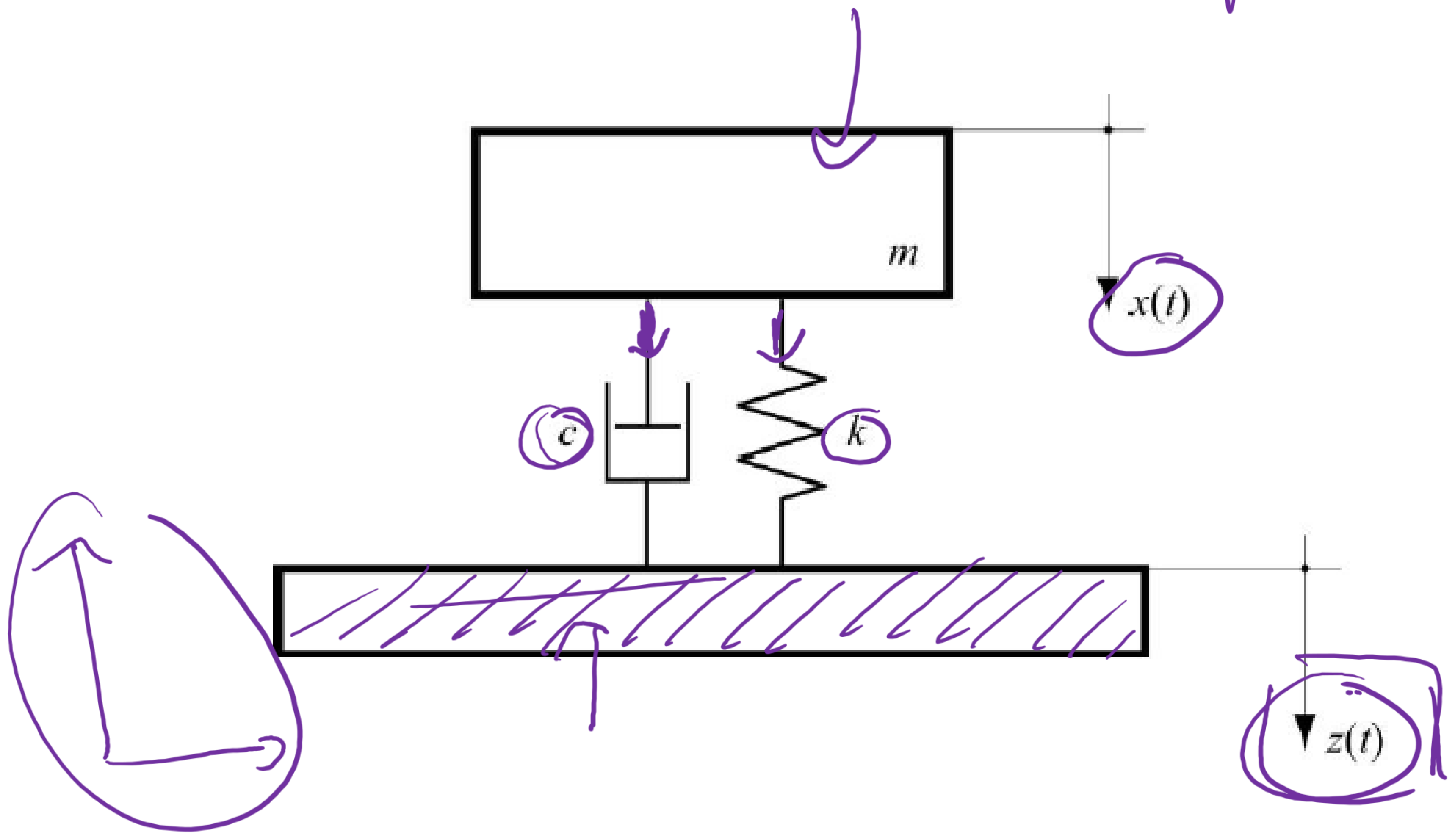
École
polytechnique
fédérale
de Lausanne

Exo 3.4



$$m\ddot{x} + c\dot{x} + kx = F_0 \cdot \cos(\omega t)$$

$$\tilde{F} = F_0 \cdot e^{-j\omega t} \quad ; \quad \tilde{x} = X_0 \cdot e^{-j\omega t}$$



$$z(t) = Z_0 \cdot \cos(\omega t)$$

$$m\ddot{x} = -k(x-z) - c(\dot{x}-\dot{z}) \Rightarrow m\ddot{x} + c\dot{x} + kx = c\dot{z} + kz$$

$$\left.\begin{aligned} \tilde{z} &= Z_0 \cdot e^{-j\omega t} \\ \tilde{x} &= X_0 \cdot e^{-j(\omega t+\varphi)} \end{aligned}\right\} \Rightarrow x(t) = X_0 \cdot \cos(\omega t - \varphi)$$

$$-\omega^2 m X_0 e^{-j(\omega t-\varphi)} + j\omega c X_0 e^{-j(\omega t-\varphi)} + k X_0 e^{-j(\omega t-\varphi)} = (j\omega c + k) Z_0 \cancel{e^{-j\omega t}}$$

$$X_0 \cdot e^{+j\varphi} (k - m\omega^2 + j\omega c) = Z_0 (k + j\omega c)$$

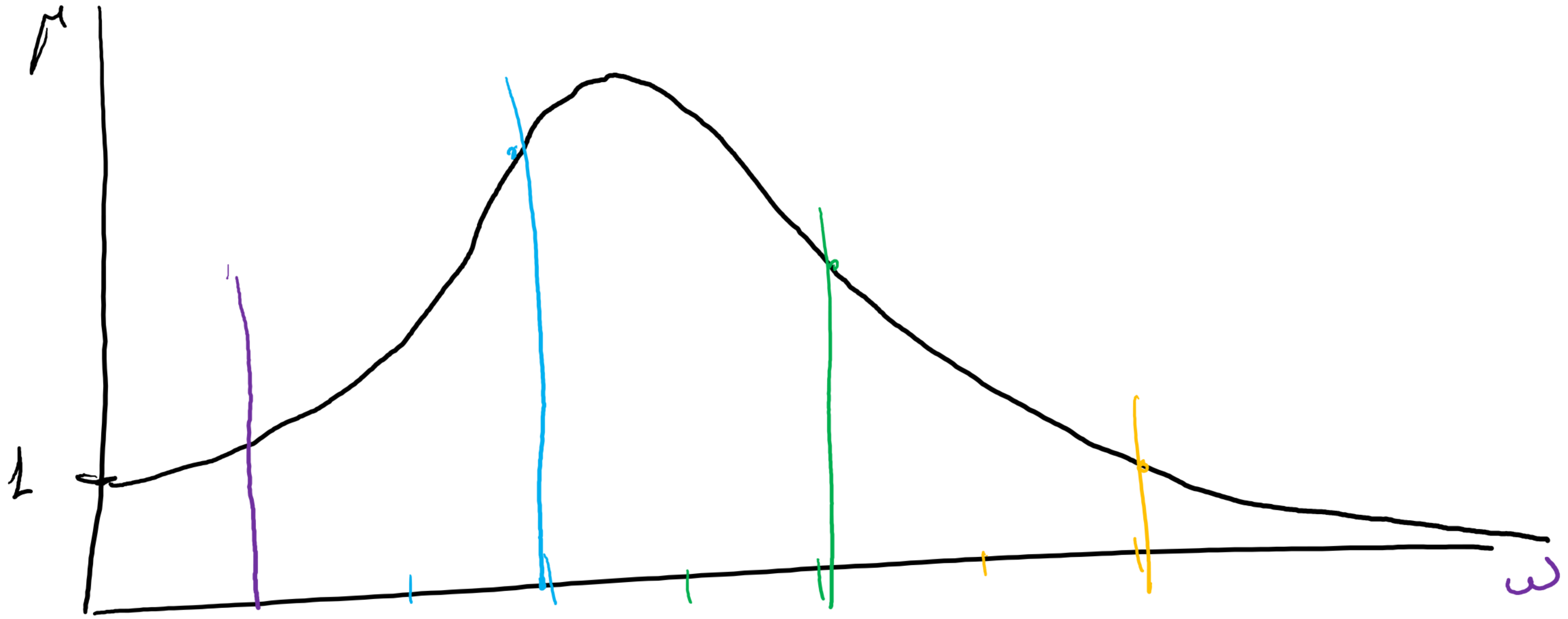
$$X_0 = Z_0 \cdot \left| \frac{k + j\omega c}{k - m\omega^2 + j\omega c} \right|$$

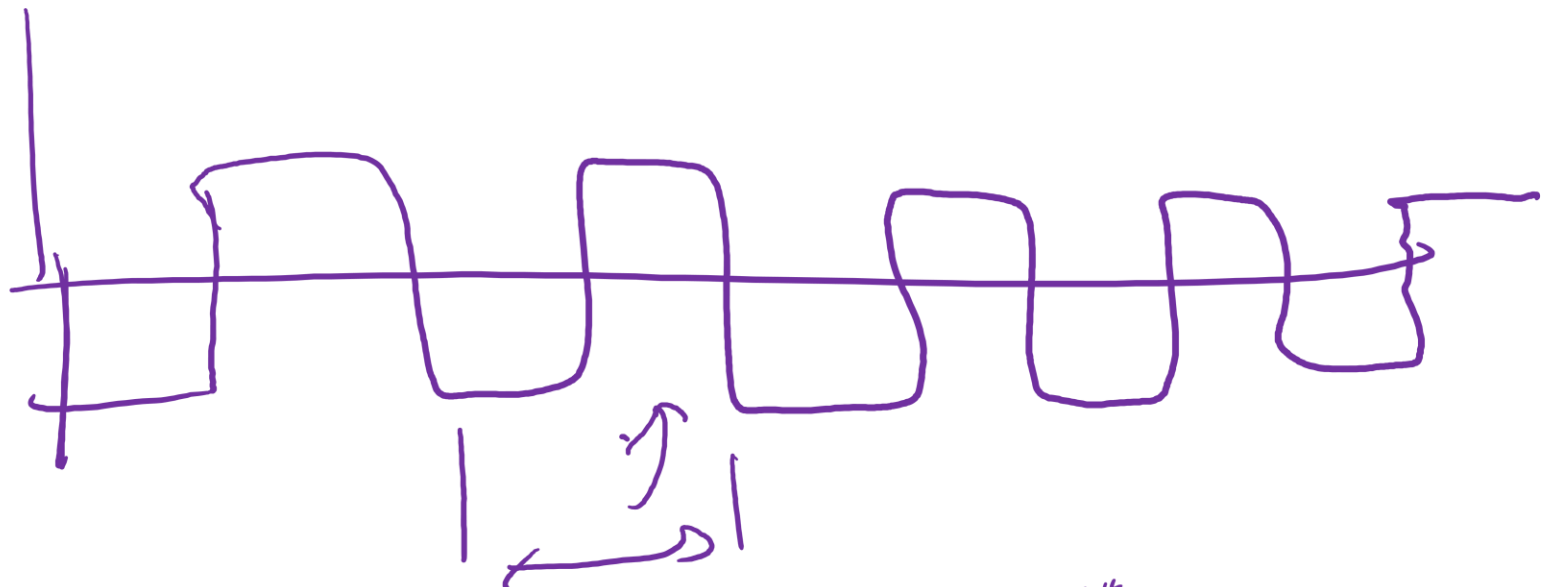
$$\varphi = \arg\left(\frac{k + j\omega c}{k - m\omega^2 + j\omega c} \right)$$

Semaine 4 - Harmoniques

Fonc. harmonique → $\propto \cos(\omega t)$ ou $\sin(\omega t)$

Fonc. périodique $= \sum_{n=1}^{\infty} A_n \cos(n \omega t) + B_n \sin(n \omega t)$





$T \rightarrow \omega = \frac{2\pi}{T}$

- $\cos \omega t$ →
- $\cos(3\omega t)$ →
- $\cos(5\omega t)$ →
- $\cos(7\omega t)$ →
- ⋮

$$c_0 = \frac{2 m \omega_0}{100}$$

$$\eta = \frac{c_0}{2 m_0 \omega_d} = \frac{1}{100}$$

$$f(t) = F_1 \cdot \cos(\omega t) + F_2 \cos(2\omega t) + F_3 \cos(3\omega t) + \dots$$

$$x(t) = X_1 \cdot \cos(\omega t + \varphi_1) + X_2 \cos(2\omega t + \varphi_2) + X_3 \cdot \cos(3\omega t + \varphi_3)$$

$$X_3 = \frac{F_3}{k} \cdot \mu$$

Semaine 4
QCMs

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Plan du cours

Week	Jours	Partie du cours	Exercices	Homework
1	09.09//11.09	Intro et oscillateur 1D libre conservatif	Série 1	Leçon 1.1&1.2
2	18.09	Oscillateur 1D libre dissipatif	Série 2	Leçon 2.1&2.2
3	23.09//25.09	Régime permanent harmonique (complexe)	Série 3	Leçon 3.1&3.2
4	30.09//02.10	Régime permanent périodique (série de Fourier)	Série 4	Leçon 4.1&4.2
5	07.10//09.10	Régime forcé général (transf. Laplace ou Fourier)	Série 5	Leçon 5.1&5.2
6	14.10//16.10	Systèmes à 2 DDL conservatifs	Série 6	Leçon 6.1&6.2
BREAK!!!				
7	28.10//30.10	Récapitulatif // séance de QCMs	MIDTERM	
8	04.11//06.11	Systèmes à 2DDL dissipatifs, osc. de Frahm	Série 7	Leçon 7.1&7.2
9	11.11//13.11	Systèmes à n DDL, conservatif	Série 8a	Leçon 8.1&8.2
10	18.11//20.11	Systèmes à n DDL, conservatif	Série 8b	
11	25.11//27.11	Systèmes à n DDL, dissipatif	Série 9	Leçon 9.1&9.2
12	02.12//04.12	Systèmes à n DDL, forcé	Série 10	Leçon 10.1&10.2
13	09.12//11.12	Systèmes continus: barres & poutres	Série 11	Leçon 11.1&11.2
14	16.12//18.12	Récapitulatif // séance de QCMs		

Before we start… let's connect!

- Smartphones
  - iPhone
  - Android

Download and install the app "*PointSolutions*"

pointsolutions powered by echo360

- Computers (or Windows Phones)
  - Go to www.responseware.eu

- Choose **EUROPE Server** (if asked)
- Join session – **ME332bis**
- When asked – **do not input your name. Enter as anonymous.**

EPFL $f(t) = F\cos(\omega t)\,(1+\cos(\omega t))$, harmoniques?

$\frac{1}{T}\int_0^T f(t)\cdot\cos(n\omega t)\,dt \leftarrow$

A. $F_0 = F, F_1 = 0, F_2 = F$

B. $F_0 = F, F_1 = F, F_2 = \frac{F}{2}$

C. $F_0 = \frac{F}{2}, F_1 = F, F_2 = F, \boxed{F_n = \frac{F}{n}}$

D. $F_0 = F, F_1 = F, F_3 = \frac{3F}{2}$

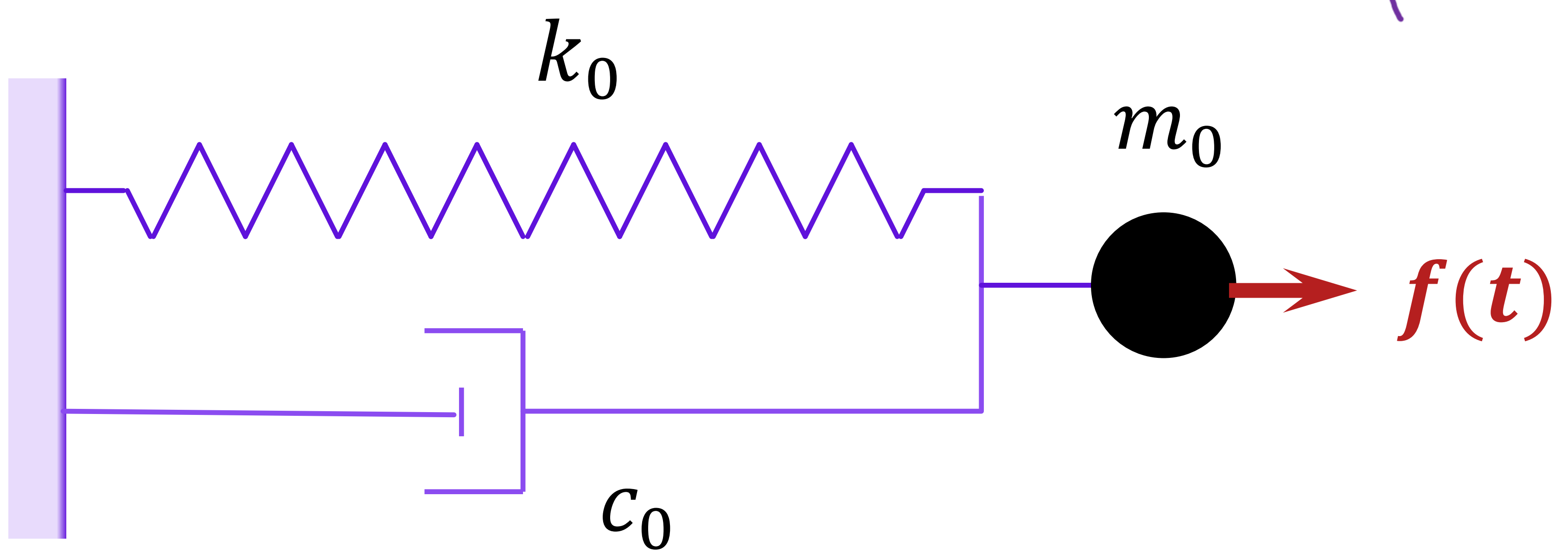
E. On ne peut pas savoir

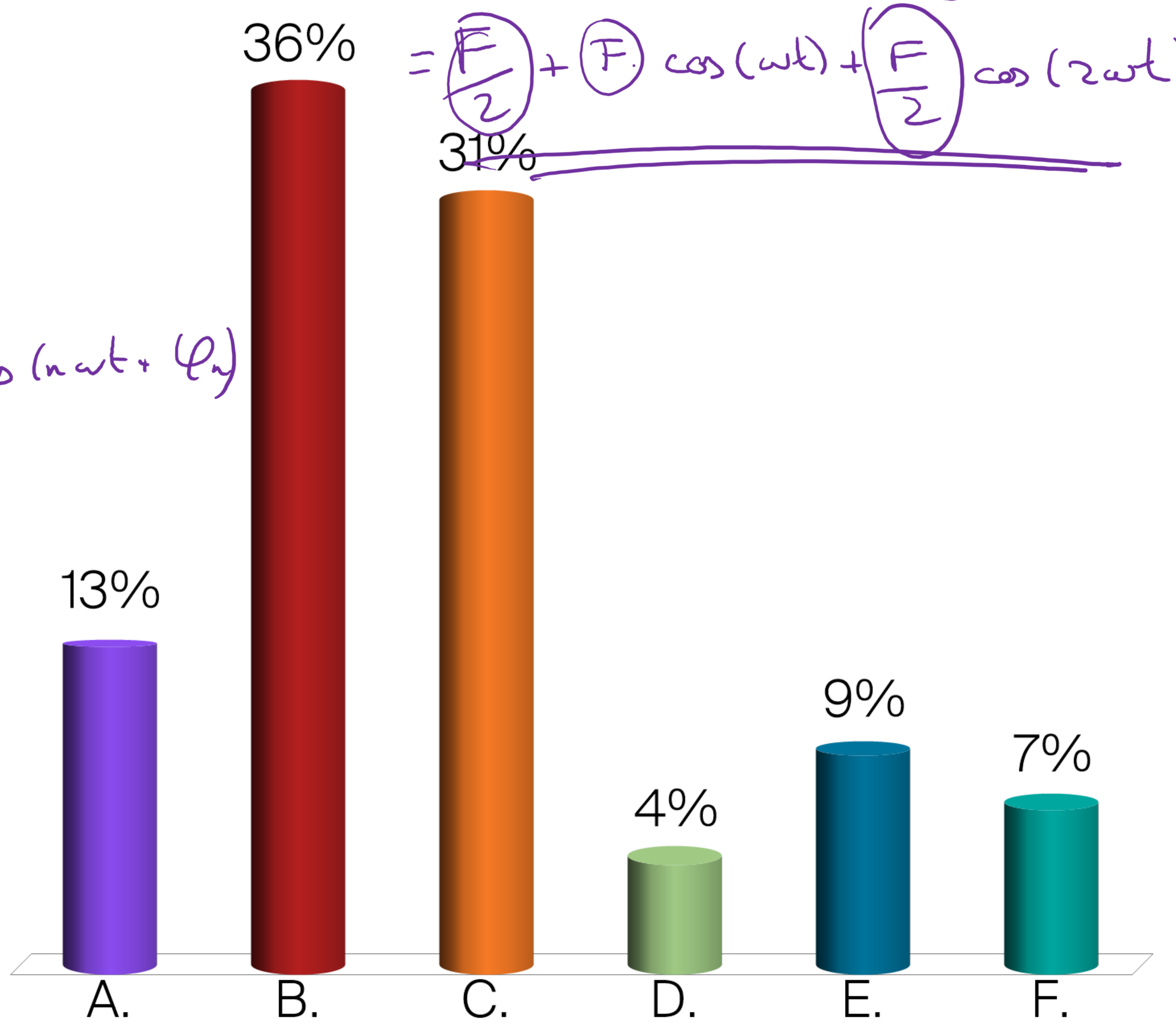
F. Trop longue

$f(t) = F\cdot\cos\omega t + F\cdot\cos^2(\omega t) = F\cdot\cos\omega t + F\left(\frac{1+\cos(2\omega t)}{2}\right)$

$= \frac{F}{2} + F\cos(\omega t) + \frac{F}{2}\cos(2\omega t)$

$f = \frac{F_0}{2} + \sum F_n\cdot\cos(n\omega t + \varphi_n)$





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EPFL À quelles ω on aura mouvement?

$f(t) = \frac{F}{2} + F\cos\omega t + \frac{F}{2}\cos 2\omega t$

$\boldsymbol{f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))}$

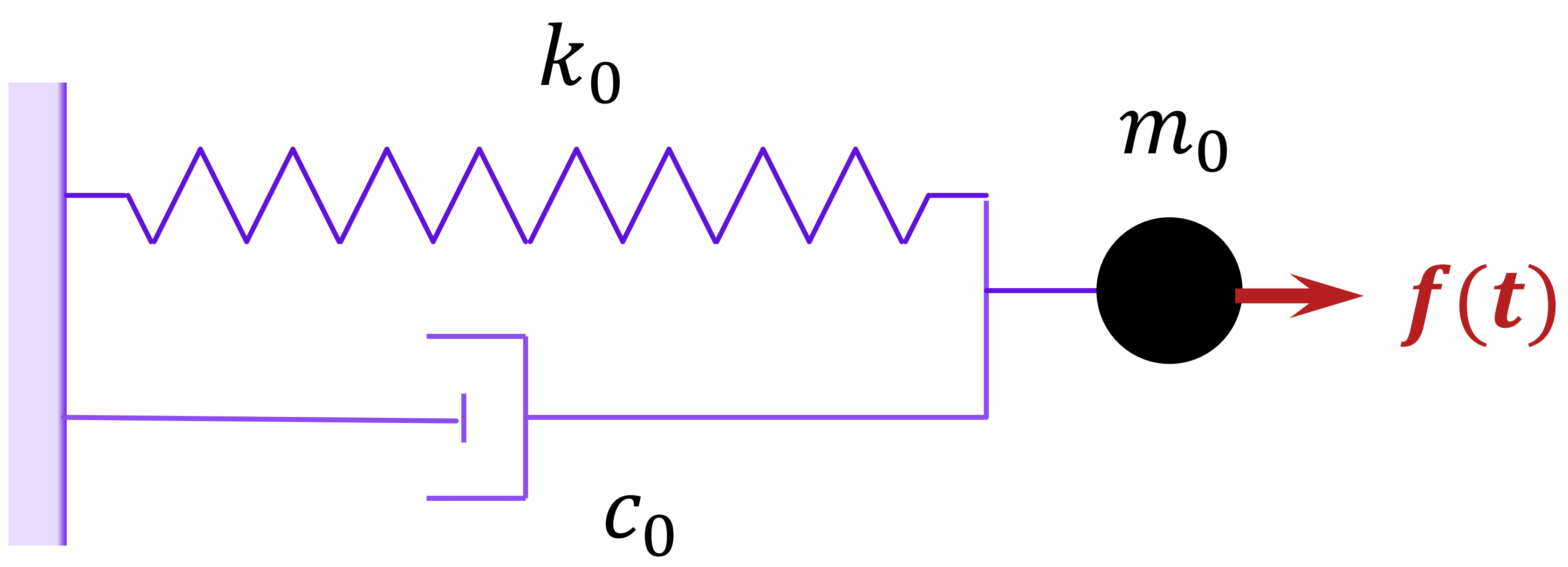
A. ω_0

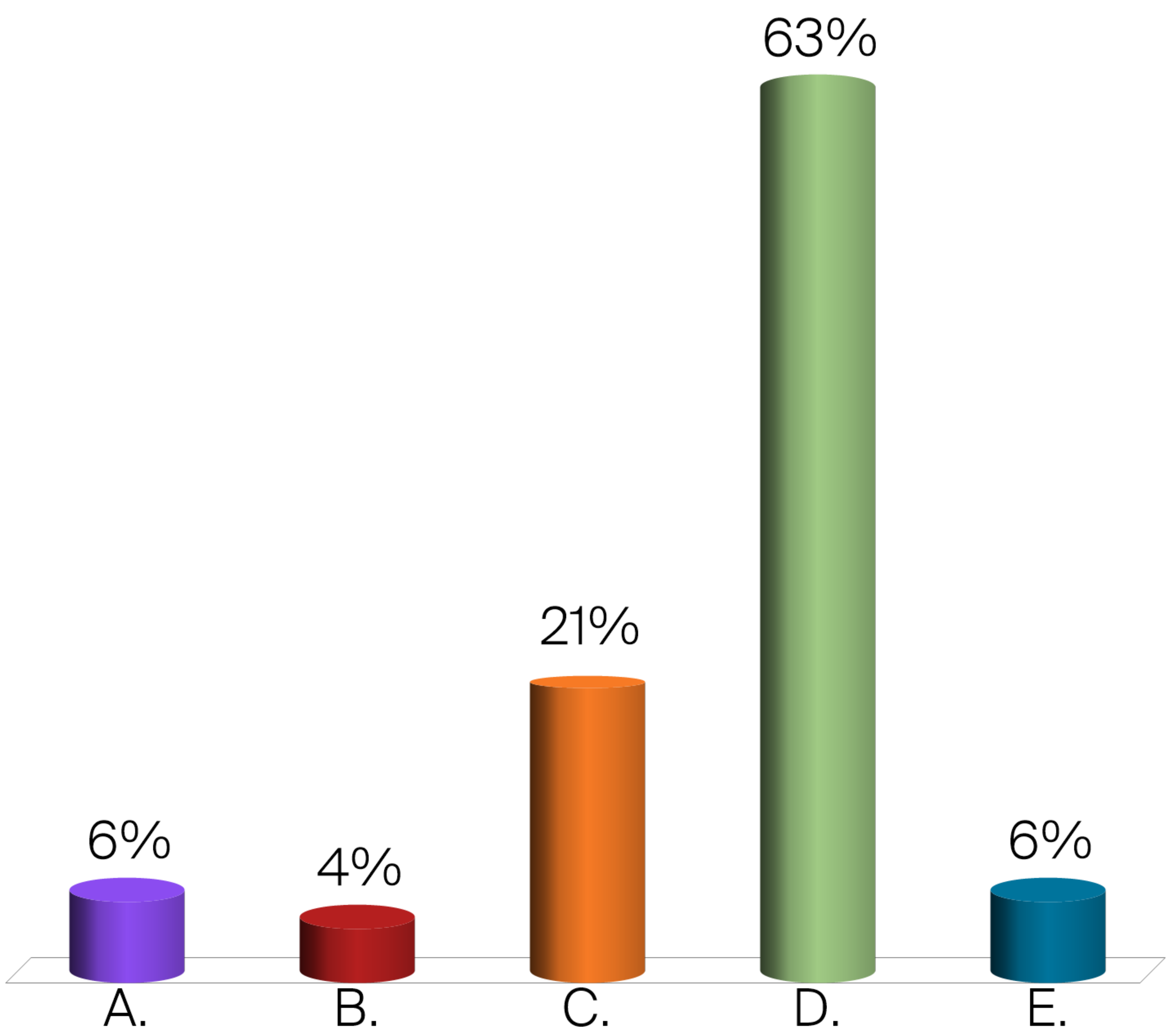
B. ω_2

C. ω

D. $\omega, 2\omega$

E. On ne peut pas savoir





 ME 332bis

Déplacement statique?

$f(t) = \frac{F}{2} + F \cos \omega t + \frac{F}{2} \cos 2\omega t$

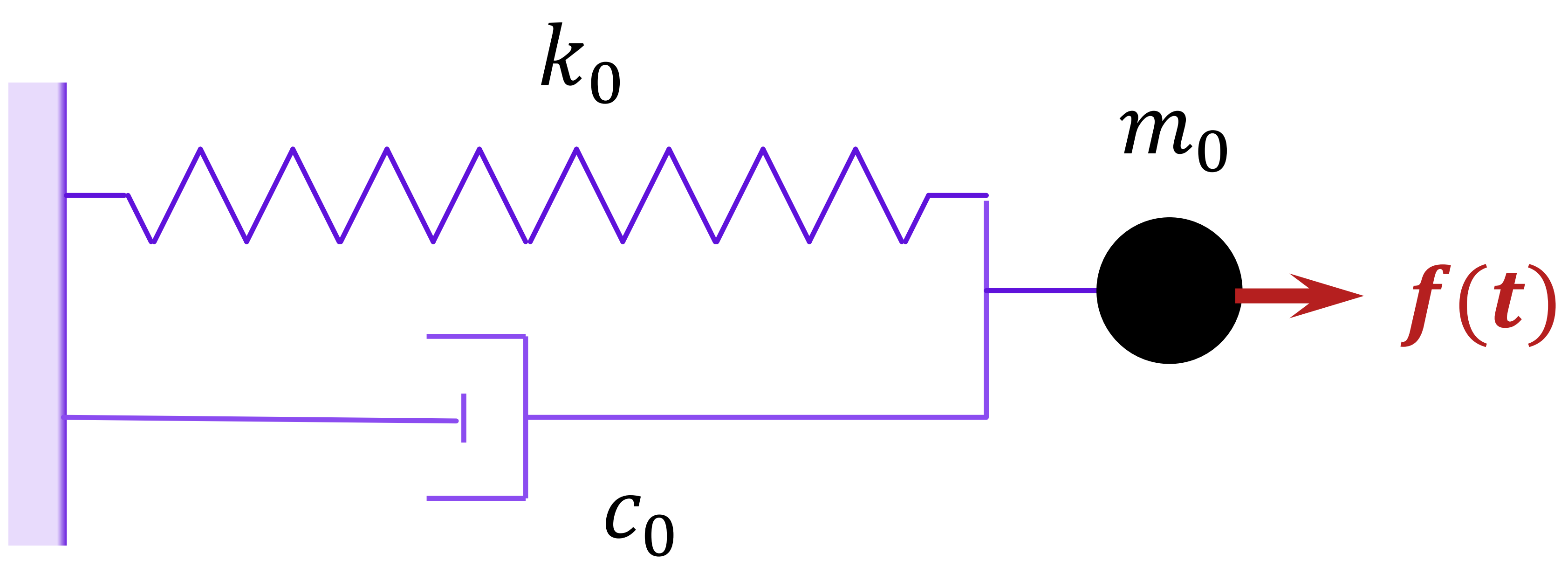
A. $\frac{F}{2}$

B. F

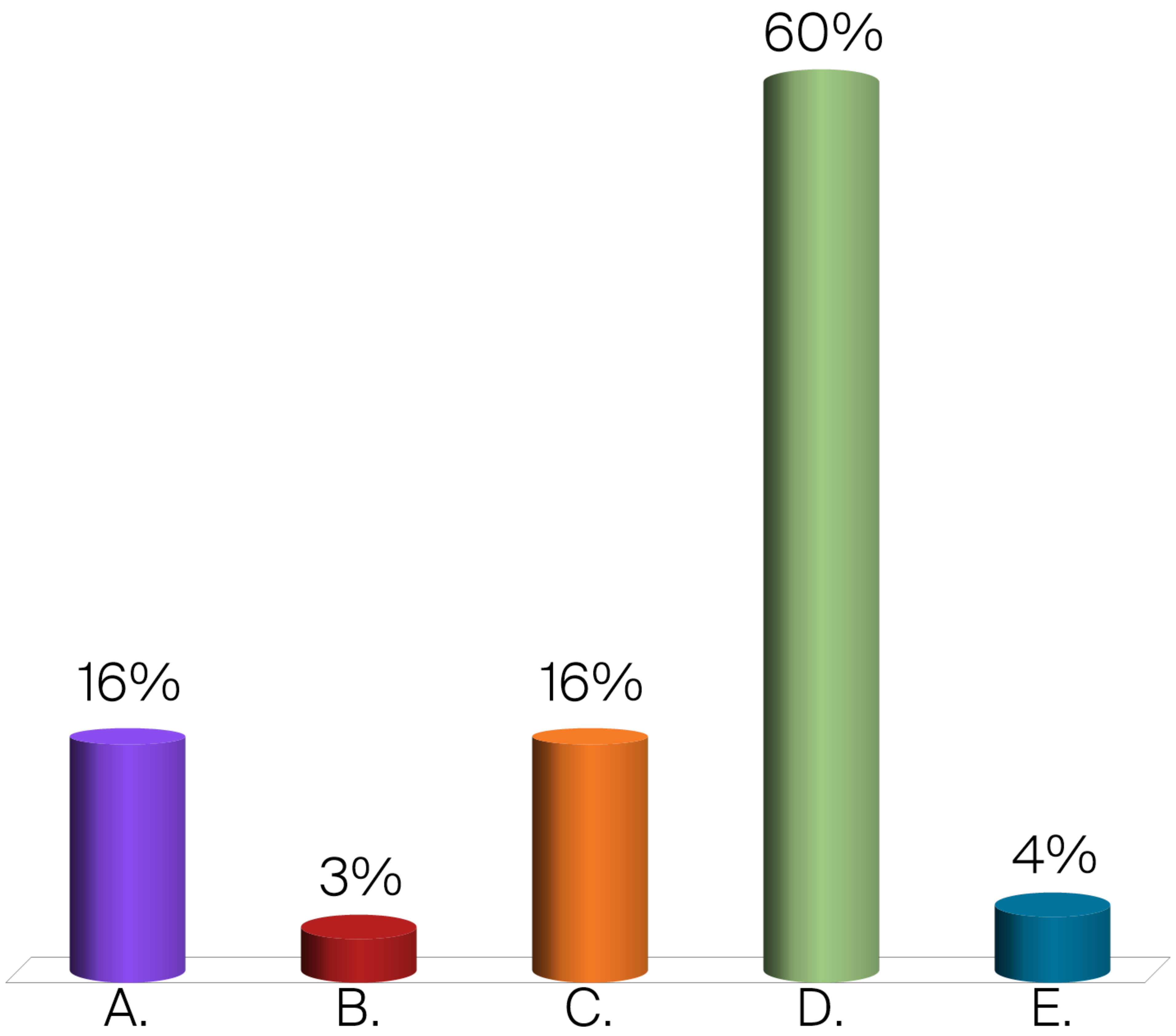
C. $\frac{F}{k_0}$

D. $\boxed{\frac{F}{2k_0}}$

E. On ne peut pas savoir



$$\boldsymbol{f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))}$$



EPFL

X_1, si $\omega = \frac{1}{2}\sqrt{\frac{k_0}{m_0}}$?

F · cos ωt

A. $\frac{F}{k_0} 100$

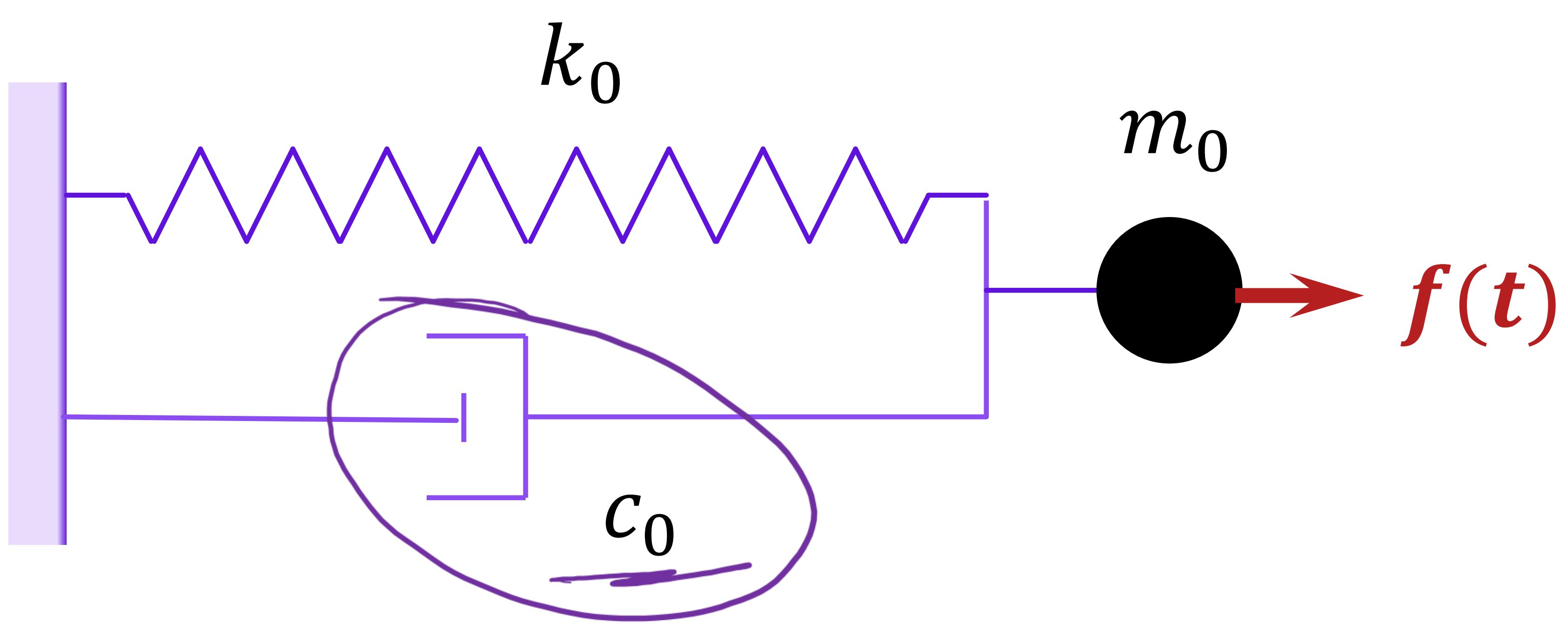
B. $\frac{F}{k_0} 0.05$

C. $\frac{F}{k_0}$

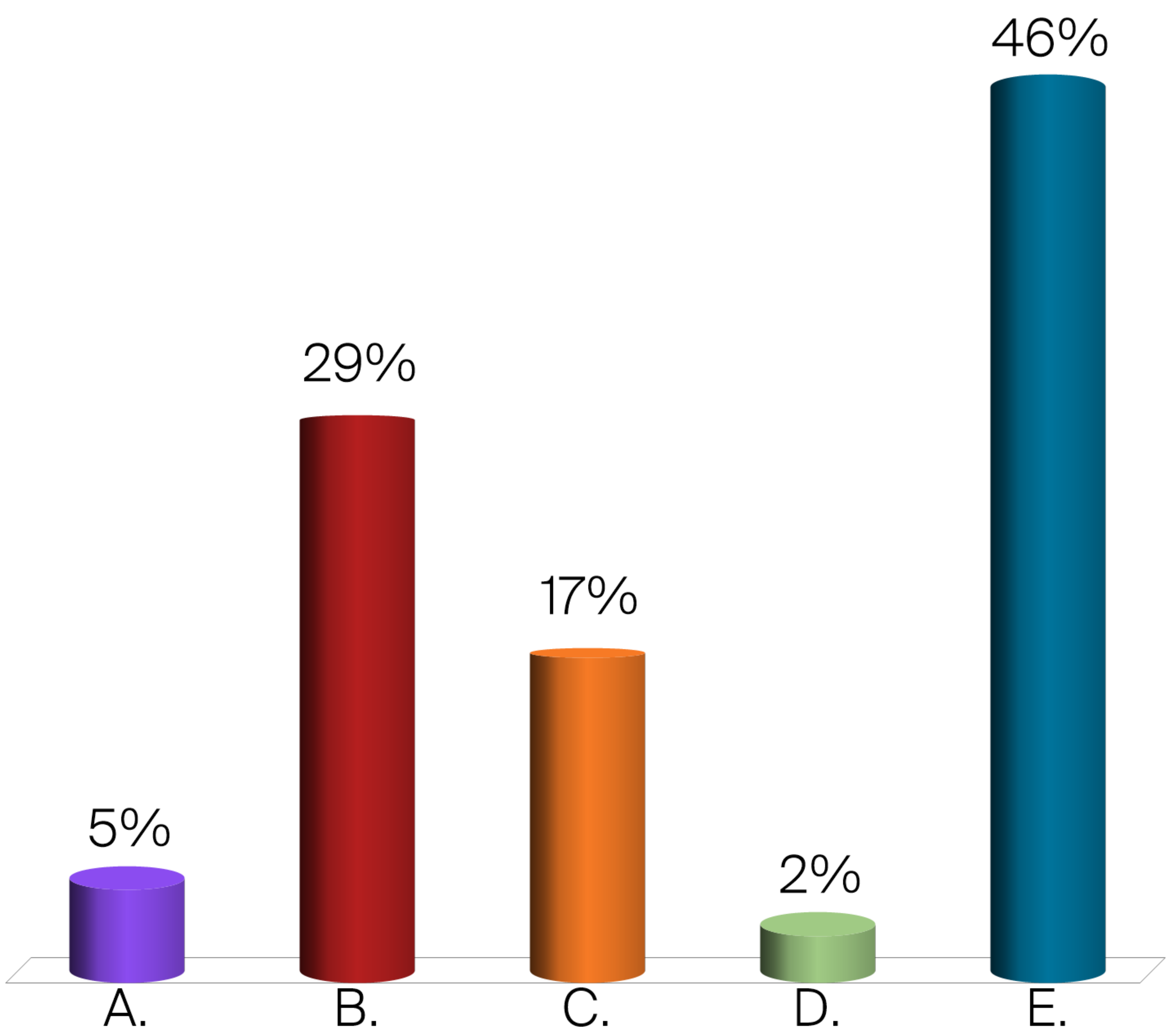
D. 0

E. On ne peut pas savoir

X



$f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))$



X_3, si $\omega = \frac{1}{2}\sqrt{\frac{k_0}{m_0}}$?

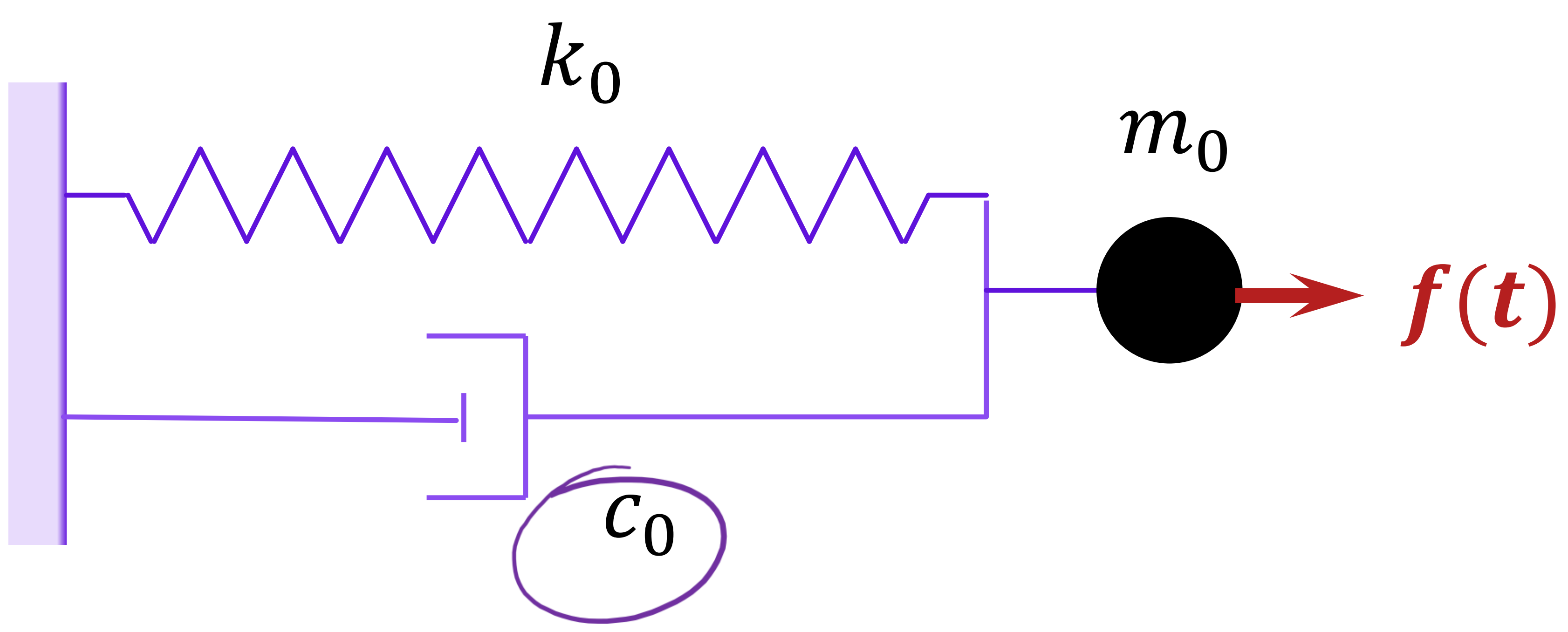
A. $\frac{F}{k_0}100$

B. $\frac{F}{k_0}0.05$

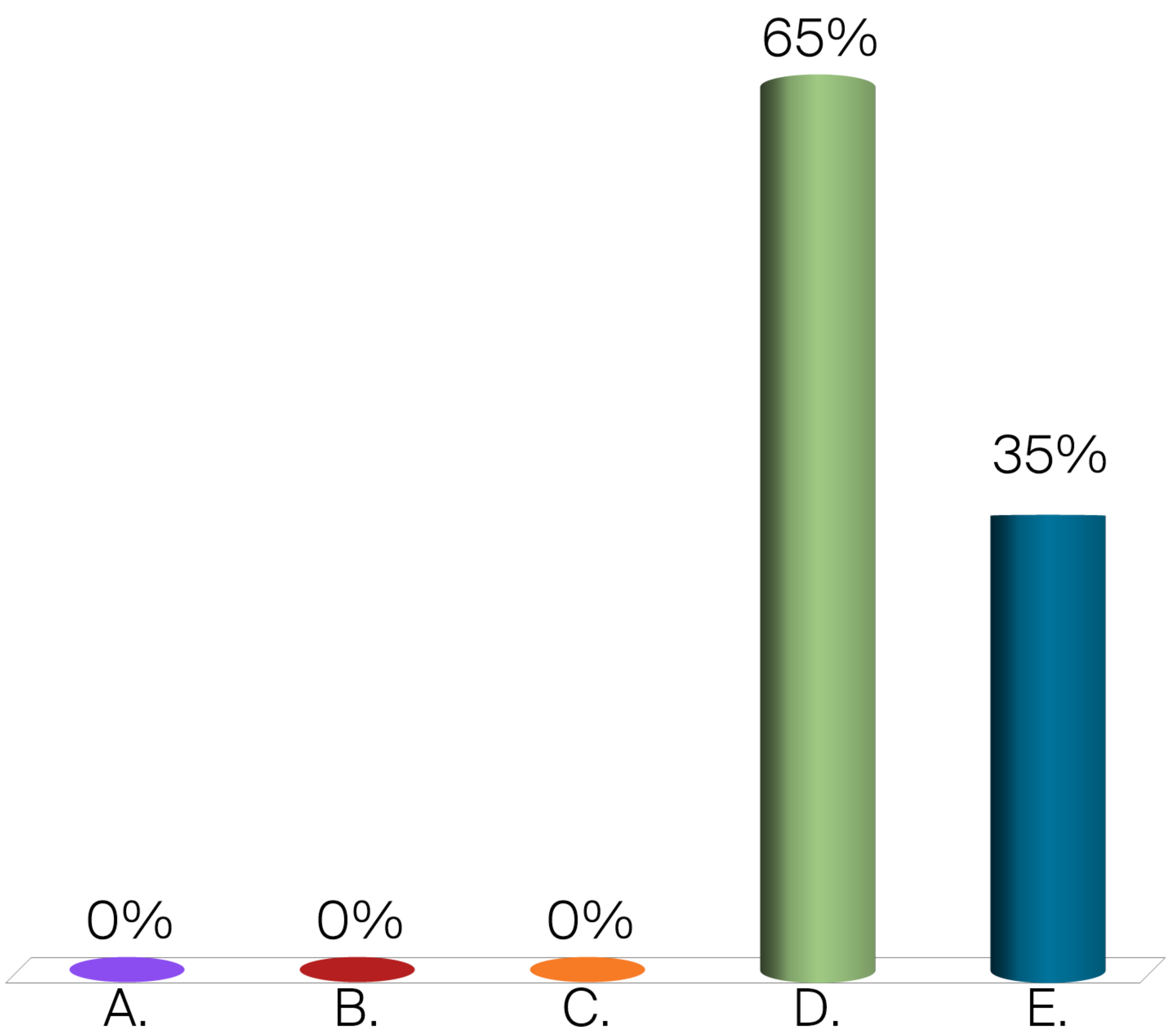
C. $\frac{F}{k_0}$

D. 0

E. On ne peut pas savoir



$$\boldsymbol{f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))}$$



EPFL

X_1, si $\omega = \frac{1}{2}\sqrt{\frac{k_0}{m_0}}$, $\eta = 0.01$?

$\omega = \frac{\omega_0}{2} \Rightarrow \beta = \frac{\omega}{\omega_s} = 1/2$

$F \cdot \cos \omega t$

A. $\frac{4}{3}\frac{F}{k_0}$

B. $\frac{16}{9}\frac{F}{k_0}$

C. 0

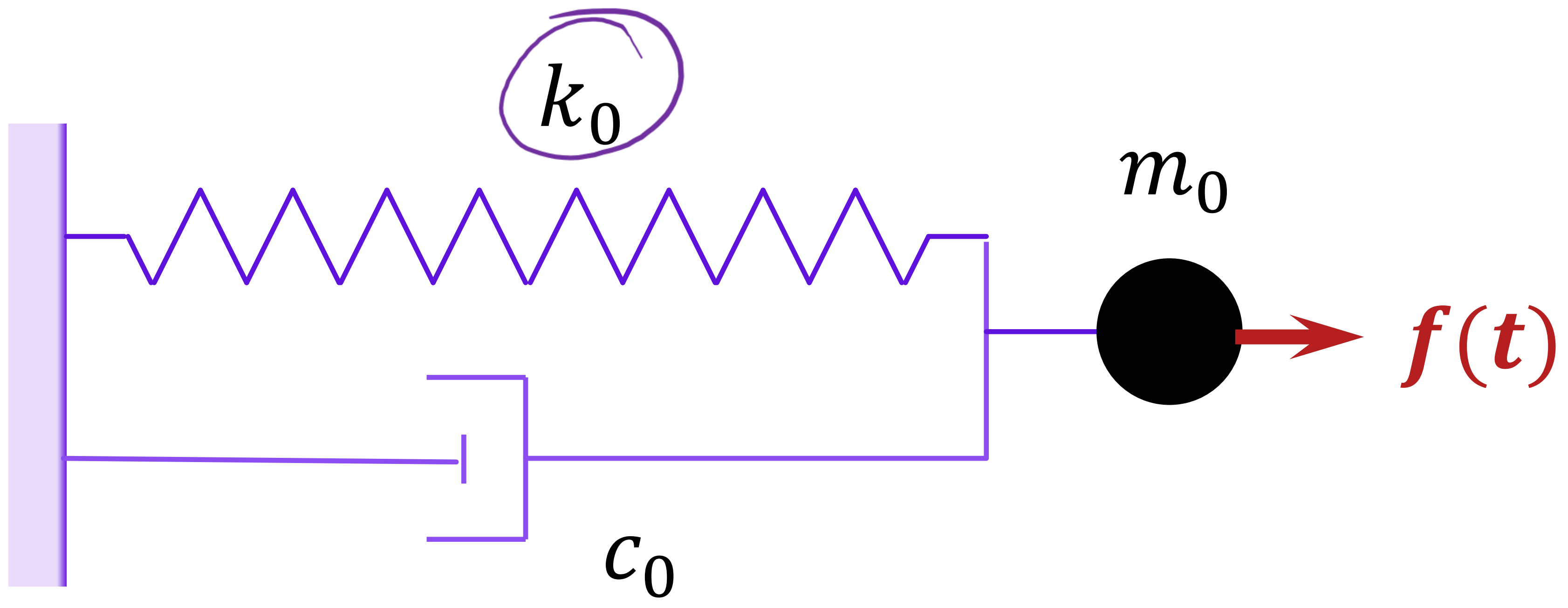
D. On ne peut pas savoir

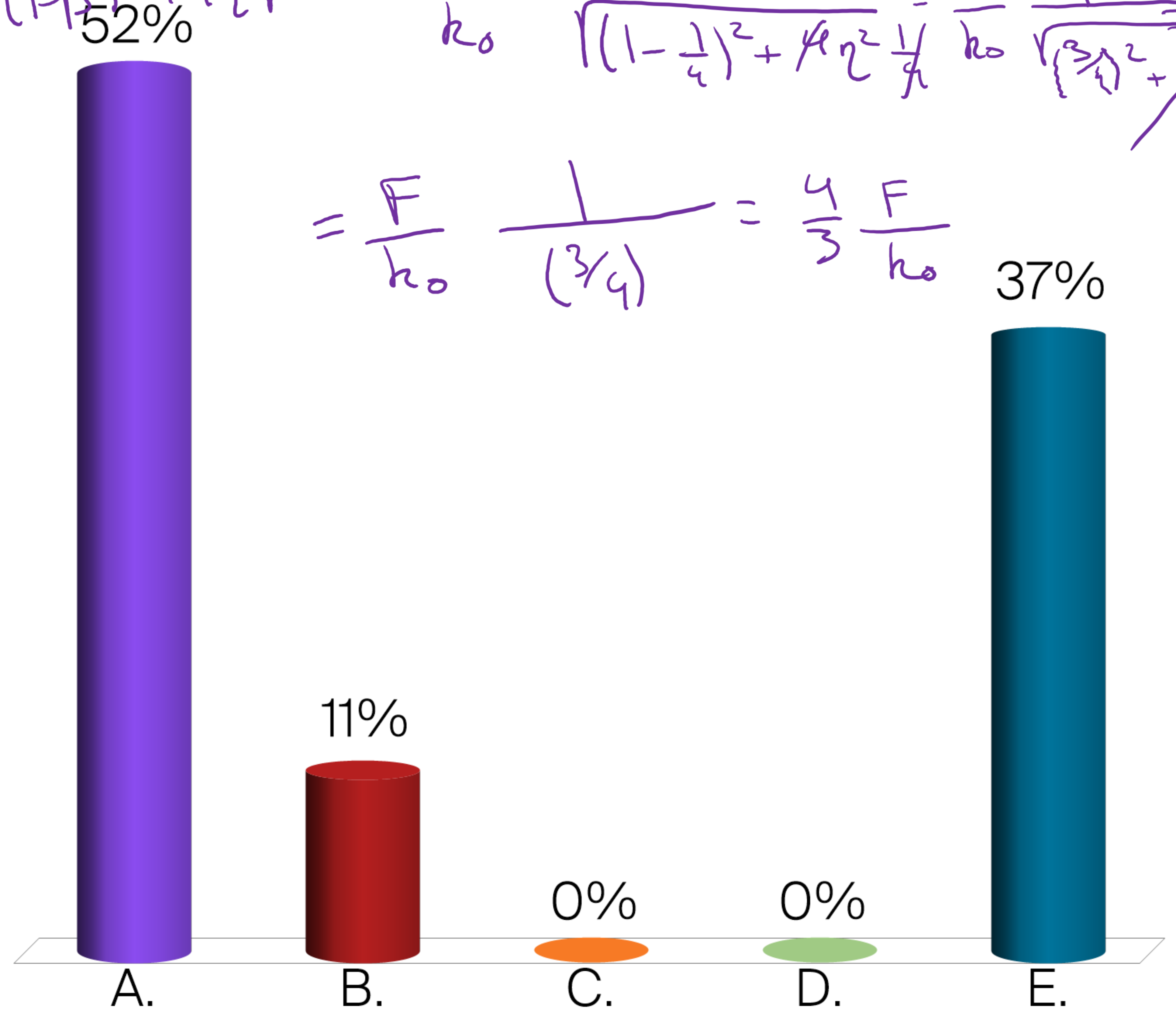
E. Aucune réponse dessus

$f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))$

$X_1 = \frac{F}{k_0} \cdot R(\omega) = \frac{F}{k_0} \cdot \frac{1}{\sqrt{(1-\beta^2)^2 + 4\eta^2\beta^2}} = \frac{F}{k_0}\frac{1}{\sqrt{(1-\frac{1}{4})^2 + 4\eta^2 \frac{1}{4}}} = \frac{F}{k_0}\frac{1}{\sqrt{(\frac{3}{4})^2 + \eta^2}}$

$= \frac{F}{k_0}\frac{1}{(3/4)} = \frac{4}{3}\frac{F}{k_0}$





X_2, si $\omega = \frac{1}{2}\sqrt{\frac{k_0}{m_0}}$, $\eta = 0.01$?

$2\omega = \omega_0 \Rightarrow \beta = 1$

$f(t) = F\cos(\omega t)\,(1 + \cos(\omega t))$

A. $100\,\frac{F}{k_0}$

B. $50\,\frac{F}{k_0}$

C. $0.01\,\frac{F}{k_0}$

D. 0

E. Aucune réponse dessus

$\frac{F}{2} \cdot \cos(2\omega t)$

$X_2 = \frac{F}{2k_0} \cdot \mu(2\omega) = \frac{F}{2k_0} \cdot \frac{1}{\sqrt{(1-1)^2 + 4\eta^2\beta^2}} = \frac{F}{4k_0}\,\frac{1}{0.01}$

$X_2 = \frac{100}{4}\,\frac{F}{k_0} = 25\,\frac{F}{k_0}$

$(2\beta)^2$